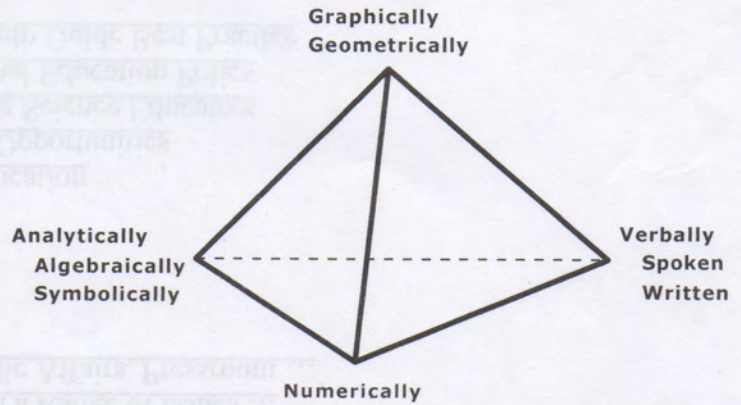


THE "RULE OF FOUR"

I Geometric Representations - Graphic Representations

- Diagrams
- Figures
- Graphs
- Models
- Pictures
- Coordinate Planes



II Numeric Representations

- Raw Data
- Lists
- Tables

III Analytic Representations

- Algebraic Representations
- Symbolic Representations

IV Verbal Representations

- Spoken-Word Explanations
 - Listening
 - Lectures
 - Classroom Discussions
 - Verbal Instructions
- Written Explanations
 - Textbooks
 - Class Notes
 - Handouts
 - Online Resources
 - Other Printed Media

THE "RULE OF FOUR"

Geometric Representations	-	Graphically
Numeric Representations	-	Numerically
Analytic Representations	-	Algebraically
Verbal Representations	-	Verbally

These four modes of representation collectively form the "Rule of Four" in mathematics education. The Harvard Calculus Consortium - the group credited with formulating the Rule of Four - lists the following as one of their guiding principles: **"Where appropriate, topics should be presented geometrically, numerically, analytically, and verbally."** The idea behind the Rule of Four is that students learn in different ways. As suggested by NCTM, all students should learn various modes of representation, but each student typically has an innate strength in one of these four areas. To ensure that the greatest number of students gain mathematical understanding, it is important to hit all four types of representations. In addition, it is important for students to develop faculty with all four types of representations.

I Geometric Representations

As the old adage states, a picture is worth a thousand words. To visual learners this statement is especially true. For those who are not primarily visual learners, **diagrams, graphs, coordinate planes, and other figures** may still add clarity.

Because visual representations appeal to everyone, not just visual learners, the first representation mentioned in the Rule of Four is the geometric representation. **A synonym might be "graphic representation."** This mode uses **pictures, diagrams, and models** to help make abstract concepts more concrete.

II Numeric Representations

With regard to the Rule of Four, using a numerical representation refers to **displaying data in an organized fashion**, possibly as an **ordered list** or in a **table**.

In the real world, data is presented numerically more often than in any other form. That's because data is initially generated as a collection of raw numbers. Only with manipulation does it become a graph, picture, algebraic equation, or paragraph.

Students need to understand numbers in their raw form. Otherwise, they will not have the flexibility to present them accurately in other arrangements. Further, some students are adept at interpreting numbers and noticing patterns among them, and for these students it is important that they have the opportunity to view numbers in their most basic state.

III Analytic Representations

The use of **symbolic notation** is so fundamental to mathematics that it is often the only representation used in traditional classrooms. To use only **algebraic representations** seems to put undue strain on students who have difficulty with symbolic manipulation.

Of course, the importance of algebraic representations should not be diminished. The language of algebra is helpful in expressing situations mathematically. Algebraic notation makes it possible to express thoughts that are cumbersome in words. Consider the verbal description of the Pythagorean theorem: "In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides." That statement is not nearly as clear or concise as $a^2 + b^2 = c^2$.

IV Verbal Representations

Verbal representations occur in two common forms: spoken and written. Teachers use both daily. Through **discussion and explanations**, teachers attempt to help their students gain understanding by engaging them in discourse about mathematics. Through **textbooks, handouts, and other printed media**, teachers attempt to develop understanding by having their students read about mathematics.

Once a problem is presented geometrically, numerically, and analytically, the teacher / students should **verbally state the conclusion to the problem.**

The above information was extracted & modified from:

<http://www.learner.org/channel/workshops/algebra/workshop5/teaching.html#1>

<http://www19.homepage.villanova.edu/alice.deanin/courses/Mat7310/rule%20of%20four.htm>